

ANALYSIS OF SINGLE AND COUPLED STRIPLINES WITH ANISOTROPIC SUBSTRATES

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ABSTRACT

The characteristic impedances and the normalized phase velocities of single and parallel-coupled striplines using anisotropic substrates were obtained by means of an integral-equation approach and a mapping method. A quasi-TEM approximation based on Laplace's equation was used. Calculated values for sapphire and pyrolytic boron nitride are given.

Introduction

The use of anisotropic substrates in microwave integrated circuits (MIC) has been investigated in the last few years. Some of the effects resulting from the utilization of these substrates in microstrip lines have already been determined and reported in the literature¹⁻³.

Using mapping Horno³ has characterized single microstrip lines with results more precise than those previously obtained by Alexopoulos and Krown². These authors used the integral-equation method⁵ with a Green's function obtained from the line-of-charge model.

In the present paper the effect of the anisotropic substrates in striplines is considered. The mapping⁴ and the integral-equation method⁵ with the elementary-strip model⁶ were used to characterize the single and parallel-coupled striplines. These are two distinct methods and were used to verify the accuracy of the data.

In the first method the structure containing an anisotropic substrate is transformed into an equivalent structure with an isotropic substrate as suggested by Horno³. Thus, single striplines in symmetric or asymmetric structures⁷⁻⁹ and parallel-coupled striplines^{9,10} can be analyzed and the efficiency of this analysis is determined by the choice of the method used to characterize the isotropic structure.

In the second method the Green's function for the elementary-strip model is first expressed in a series form using the theory of residues. The characteristics of the single and parallel-coupled striplines are then obtained using the method of moments.

Theoretical Analysis

The theory described here is applicable to single and parallel-coupled symmetric striplines (see Fig. 1). The thickness of the conducting strip is considered small and is neglected. The dielectric is anisotropic with a diagonal relative-permittivity tensor and having its optical axis along the y-direction. The analysis is developed for a TEM-wave and uses the Green's function approach and the mapping method.

For single striplines the integral form of the Green's function in $y=h$ (see Fig. 1a) was obtained for the elementary-strip model as

$$\Phi(x, h) = \frac{1}{4\pi\epsilon_0 n_x n_y} \int_{-\infty}^{\infty} \frac{\cos(kx/n_x)}{\coth(kh/n_y)} \cdot \frac{\sin(ka/2n_x)}{ka/2n_x} dk \quad (1)$$

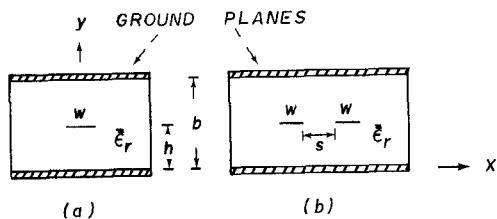


FIGURE 1: (a) SINGLE AND (b) PARALLEL-COUPLED STRIPLINES

where $n_x^2 = \epsilon_x$, $n_y^2 = \epsilon_y$, and a is the width of the elementary strip.

The line capacitance per unit of length was obtained by the integral-equation method with the method of moments^{5,6}. The method of residues was used to integrate $\Phi(x, h)$.

For the parallel-coupled striplines the Green's functions for the even and odd modes were obtained from (1), as used by Bryant and Weiss⁶.

The solution of electrostatic problems in anisotropic media using mapping between two regions R_z and R_w in the complex planes $z=x+jy$ and $w=u+jv$, respectively, was considered in a general way by Kusase and Terakado⁴.

Using mapping the anisotropic structures of interest here are transformed into other isotropic ones with the following parameters

$$\epsilon'_r = \sqrt{\epsilon_x \epsilon_y} \quad (2)$$

$$b' = b \sqrt{\epsilon_x / \epsilon_y} \quad (3)$$

$$W' = W \quad (4)$$

$$s' = s \quad (5)$$

Since the capacitance per unit of line-length is not changed with the transformation from the z - to the w -plane, the characteristic impedance of the stripline with anisotropic substrate, Z_0 , may be expressed as a function of the same impedance of the equivalent isotropic stripline, Z'_0 , as

$$Z_0 = Z'_0 \sqrt{Z_v / Z'_v} \quad (6)$$

where Z_v and Z'_v are the values of Z_0 and Z'_0 , respectively, when the dielectric is free space.

For single striplines with isotropic substrates having a dielectric constant, ϵ'_r , the characteristic impedance can be evaluated from⁷

$$Z_0' = \frac{30}{\sqrt{\epsilon_r}} \ln \left\{ 1 + \frac{1}{2} \left(\frac{16h'}{\pi W'} \right) \left[\frac{16h'}{\pi W'} \right] \right. \\ \left. + \sqrt{\left(\frac{16h'}{\pi W'} \right)^2 + 6.27} \right\} \quad (7)$$

and the wavelength of the stripline with anisotropic substrate, λ , normalized with respect to the free-space wavelength, λ_0 , is given by

$$\frac{\lambda}{\lambda_0} = \frac{Z_0}{Z_v} = \frac{v_p}{c} \quad (8)$$

where v_p is the phase velocity of the stripline with anisotropic substrate and c is the velocity of light in free space.

For the case of parallel-coupled striplines (see Fig. 1.b) (6) and (8) are valid. However (7) should be replaced by the available expressions of the characteristic impedances for the even and odd modes, Z_{oe} and Z_{oo} respectively^{9,10}.

Numerical Analysis

Using the theory of residues the Green's function given by (1) was written in a series form that converges for all values of x . The numerical values obtained by means of this Green's function were introduced in a computer program based on the method of moments. The capacitance per unit of line-length and other characteristics were then obtained for single and parallel-coupled striplines.

The same characteristics were calculated by means of another computer program based on the mapping method, using (2)-(8) and the expressions by Cohn^{9,10}. Elliptic integrals were calculated using the SSP/IBM subroutines.

Results

Two different materials were considered for the calculation of the characteristic impedances and the normalized phase velocities for the structures of Fig. 1. One of them consisted of sapphire ($\epsilon_x = \epsilon_z = 9.4$; $\epsilon_y = 11.6$) and the other was the pyrolytic boron nitride ($\epsilon_x = \epsilon_z = 5.12$; $\epsilon_y = 3.4$).

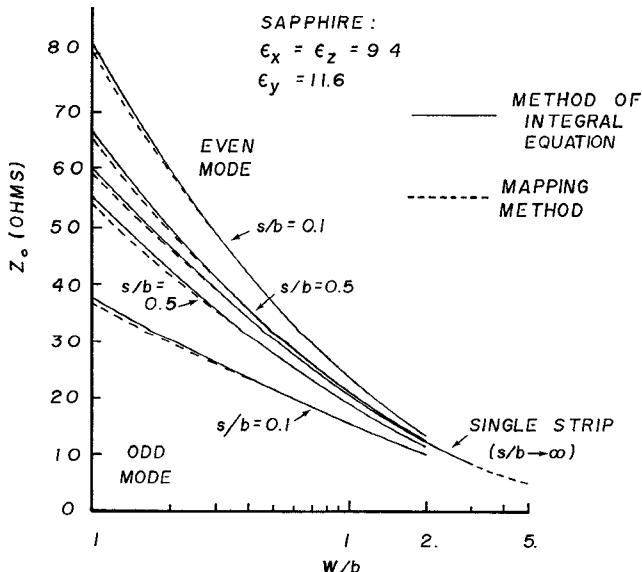
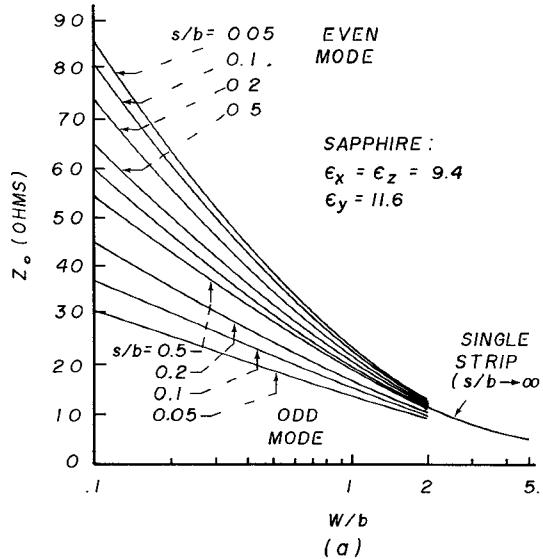


FIGURE 2: CHARACTERISTIC IMPEDANCES VERSUS W/b FOR STRIPLINES WITH SAPPHIRE SUBSTRATE.

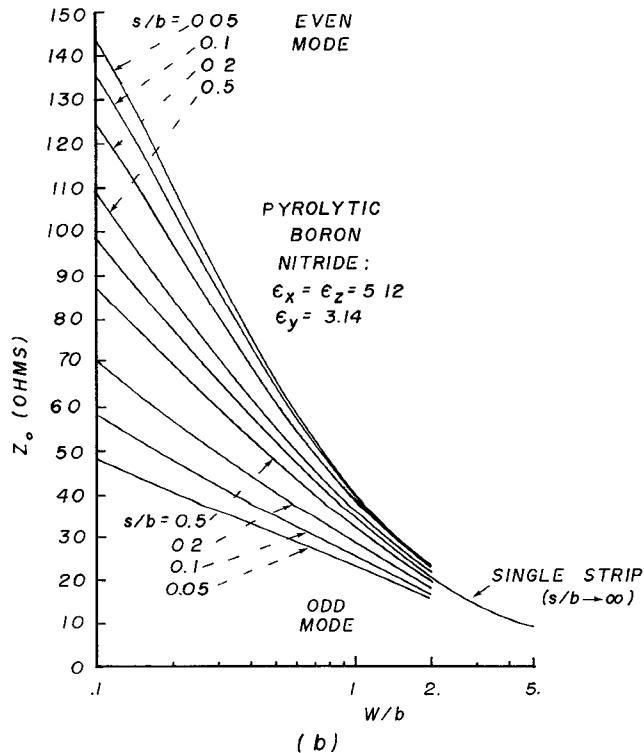
For single striplines the characteristics were evaluated for $0.1 \leq W/b \leq 5.0$. For parallel-coupled striplines characteristics of the even and odd modes were obtained for $s/b = 0.05, 0.1, 0.2$ and 0.5 and for $0.1 \leq W/b \leq 2.0$.

Fig. 2 shows the characteristic impedances for single and parallel-coupled striplines with sapphire as functions of W/b and for various values of s/b . Results from the moments approach and from the mapping method are presented.

Fig. 3 shows the characteristic impedances for single and parallel-coupled stripline as functions of W/b for (a) sapphire and (b) pyrolytic boron nitride. Fig. 4 shows the phase velocities normalized with respect to their values in free space as functions of W/b and for the same substrates.



(a)



(b)

FIGURE 3: CHARACTERISTIC IMPEDANCES VERSUS W/b FOR (a) SAPPHIRE AND (b) PYROLYTIC BORON NITRIDE.

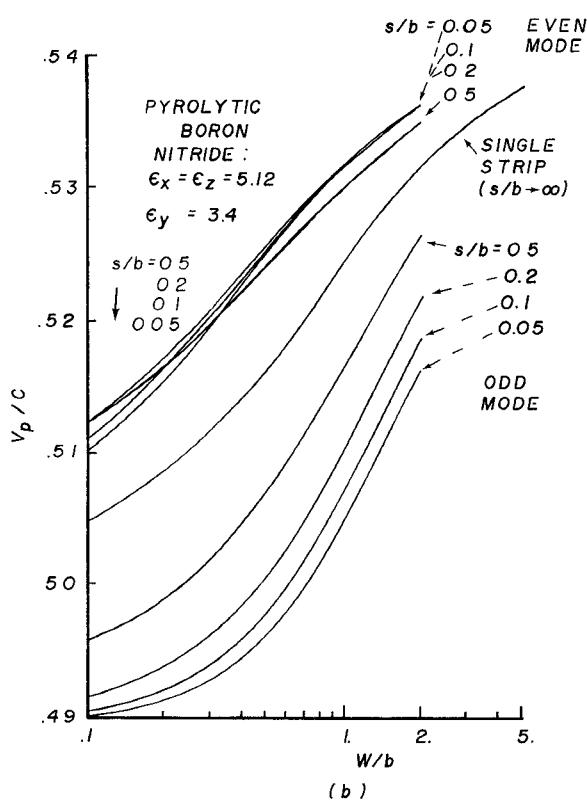
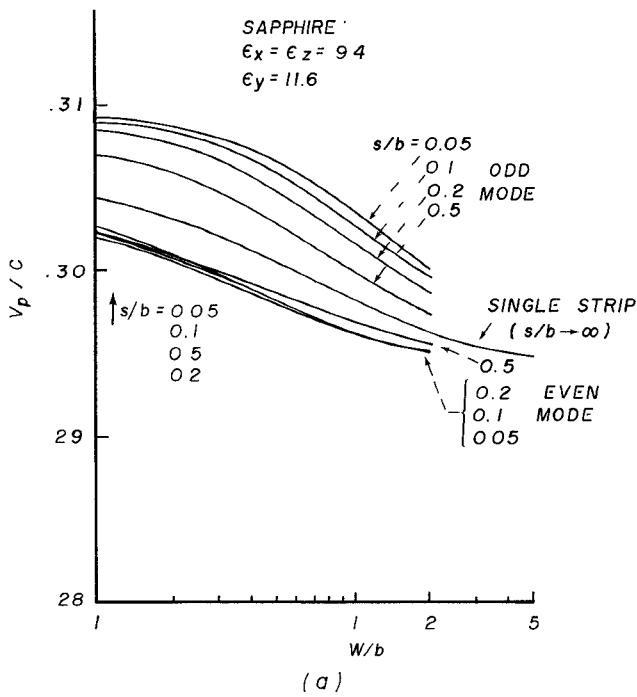


FIGURE 4: NORMALIZED PHASE VELOCITIES VERSUS W/b FOR
(a) SAPPHIRE AND (b) PYROLYTIC BORON NITRIDE.

Conclusions

From Fig. 2 the agreement between the moments approach and the mapping method is evident. The differences between the two methods decrease as W/b increases. The accuracy of the moments approach can be increased with the decrease of the width of the elementary-strip.

One should note, from Fig. 4, that the phase velocities for the even and odd modes differ as ϵ_y/ϵ_x deviates from unity. This suggests that the performance of couplers using parallel-coupled striplines is deteriorated when anisotropic substrates are used.

Acknowledgements

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